

## Home Work of Week 7

**Deadline: 9:00am, November 29(Thursday), 2018**

1. Recall that  $\mathcal{G}_n$  is the uniformly distributed  $n$ -vertex random graph, and that  $\mathcal{G}_{n,p}$  is the  $n$ -vertex random graph each of whose edge appears independently with probability  $p$ . Prove that  $\mathcal{G}_n$  and  $\mathcal{G}_{n, \frac{1}{2}}$  are identically distributed.
2. (**Bonus score 5 points**) We know that  $\lim_{n \rightarrow \infty} \Pr(\mathcal{G}_{n,p} \text{ has an isolated vertex}) = 1 - e^{-e^{-c}}$  when  $p = \frac{\ln n + c}{n}$ . Based on this fact, prove that  $\lim_{n \rightarrow \infty} \Pr(\mathcal{G}_{n,m} \text{ has an isolated vertex}) = 1 - e^{-e^{-c}}$  when  $m = \frac{n \ln n + cn}{2}$ . (Hint: it may be helpful to follow the basic idea in proving the similar result of coupon collector problem.)
3. Do Bernoulli experiment for 20 trials, using a new 1-Yuan coin. Record the result in a string  $s_1 s_2 \dots s_i \dots s_{20}$ , where  $s_i$  is 1 if the  $i^{\text{th}}$  trial gets Head, and otherwise is 0.