Homework of Week 4

Deadline: 9:00am, November 8(Thursday), 2018

1. Suppose that balls are thrown randomly into n bins. Show, for some constant c_1, that if there are $c_1\sqrt{n}$ balls then the probability that no two land in the same bin is at most $1/e$. Similarly, show for some constant c_2 (and sufficiently large n) that, if there are $c_2\sqrt{n}$ balls, then the probability that no two land in the same bin is at least $1/2$. Make these constants as close to optimum as possible. Hint: you may need the fact that $e^{-x} \geq 1 - x$ and $e^{-x-x^2} \leq 1 - x$ for $x \leq 1/2$.

2. Let X be a Poisson random variable with mean μ, representing the number of errors on a page of this book. Each error is independently a grammatical error with probability p and a spelling error with probability $1 - p$. If Y and Z are random variables representing the numbers of grammatical and spelling errors (respectively) on a page of this book, Prove that Y and Z are Poisson random variables with means $p\mu$ and $(1 - p)\mu$, respectively. Also, prove that Y and Z are independent.

3. There are n students in a classroom. Assume that their birthdays are uniformly randomly distributed and that every year has 365 days. Calculate the probability that there are two students having the same birthday and the probability that randomly choosing a student, there exists another student having the same birthday with him/her.

4. Prove Chernoff-like bounds for Poisson random variable X_μ with expectation μ:

   (a) If $x > \mu$, then $Pr(X_\mu \geq x) \leq \frac{e^{-\mu}(e\mu)^x}{x^x}$

   (b) If $x < \mu$, then $Pr(X_\mu \leq x) \leq \frac{e^{-\mu}(e\mu)^x}{x^x}$

5. Do Bernoulli experiment for 20 trials, using a new 1-Yuan coin. Record the result in a string $s_1 s_2 ... s_i ... s_{20}$, where s_i is 1 if the i^{th} trial gets Head, and otherwise is 0.